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Realistic neutrino masses from multi-brane extensions of the Randall–Sundrum model?

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Abstract. Scenarios based on the existence of large or warped (Randall–Sundrum model) extra dimensions have been proposed for addressing the long standing puzzle of the gauge hierarchy problem. Within the contexts of both those scenarios, a novel and original type of mechanism generating small (Dirac) neutrino masses, which relies on the presence of additional right-handed neutrinos that propagate in the bulk, has arisen. The main objective of the present study is to determine whether this geometrical mechanism can produce reasonable neutrino masses also in the interesting multi-brane extensions of the Randall–Sundrum model. We demonstrate that, in some multi-brane extensions, neutrino masses in agreement with all relevant experimental bounds can indeed be generated but at the price of a constraint (stronger than the existing ones) on the bulk geometry, and that the other multi-brane models even conflict with those experimental bounds.

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1 Introduction

The old proposal for additional spatial dimension(s) [1,2] and the more recent idea of brane universe models [3–7] have received considerable attention in the late nineties as novel frameworks for addressing a long standing puzzle: the gauge hierarchy problem. Indeed, several new approaches toward the gauge hierarchy question, based on the existence of extra dimension(s), have been suggested in the literature [8–17].

The first approach [8–10], which was proposed by Arkani-Hamed, Dimopoulos and Dvali (ADD), is the following one. If spacetime is the product of a 4-dimensional Minkowski spacetime with a n-dimensional compact space, and standard model (SM) fields are confined to the 4-dimensional subspace whereas gravity propagates also in the extra compact space, then one has

$$M_{\rm Pl}^2 = M^{n+2} V_n, \tag{1}$$

M being the fundamental $(4+n)$-dimensional mass scale of gravity, $M_{\rm Pl} = 1/\sqrt{8\pi G_{\rm N}} \simeq 2.4\ 10^{18}$ GeV ($G_{\rm N} \equiv$ Newton constant) the effective 4-dimensional (reduced) Planck scale and V_n the volume of compact space. Hence, by taking a sufficiently large size of new n dimensions, the value of fundamental scale M can be of the order of TeV, which removes the important hierarchy between the gravitational and electroweak energy scales. Nevertheless, another hierarchy is then introduced: the discrepancy between the electroweak symmetry breaking scale (~ 100 GeV) and the compactification scale ($\sim V_n^{-1/n}$).

An elegant alternative solution to the gauge hierarchy problem was proposed by Randall and Sundrum (RS) [11, 12]. The RS scenario consists of a 5-dimensional theory in which the extra dimension (parameterized by y) is compactified on a $S^1/\mathbb{Z}_2$ orbifold of radius $R_{\rm c}$ (so that $-\pi R_{\rm c} \leq y \leq \pi R_{\rm c}$). Gravity propagates in the bulk and the fifth dimension is bordered by two 3-branes with tensions tuned such that,

$$\Lambda_{(y=0)} = -\Lambda_{(y=\pi R_{\rm c})} = -\Lambda/k = 24 k M_5^3, \tag{2}$$

where Λ is the bulk cosmological constant, M_5 the fundamental 5-dimensional Planck scale and $1/k$ the anti-de-Sitter (AdS_5) curvature radius (see below). Within this context, the zero mode of graviton is localized on the positive tension brane, namely the 3-brane at $y=0$. Hence, while on this brane (referred to as the Planck brane) the 4-dimensional Planck mass is of order $M_{\rm Pl}$, on the other brane (at $y = \pi R_{\rm c}$) the effective Planck scale,

$$M_\star = w M_{\rm Pl}, \tag{3}$$

is affected by the exponential "warp" factor $w = \mathrm{e}^{-k\pi R_{\rm c}}$. From (3), we see that for a small extra dimension such that $R_{\rm c} \sim 11/k$ (k is typically of order $M_5 \sim M_{\rm Pl}$), one has $w \sim 10^{-15}$ and $M_\star = \mathcal{O}(1)$ TeV (the 3-brane at $y = \pi R_{\rm c}$ is then called the TeV-brane). If the SM particles (in particular the Higgs boson) are confined to the TeV-brane, they feel an effective Planck scale $M_\star$ of the same order

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of magnitude as the electroweak scale. In this sense, the RS model provides a new natural solution to the gauge hierarchy problem.

Besides, in the RS framework, no additional strong hierarchy between fundamental scales appears (in contrast with the ADD approach) as the compactification scale $(2\pi R_c)^{-1}$ is of order $M_5/70$. However, in the RS scenario, we live on a brane with negative tension (see (2)) which seems generically problematic as far as gravity and cosmology are concerned (see [16, 18–31] for a complete discussion). In particular, as it is clear from the corrected Friedmann equation for the Hubble expansion rate of our brane world, on our negative tension brane the energy density of normal matter/radiation should be negative, which conflicts with the absence of any noticeable effects of anti-gravity in our universe.

In order to avoid this potential cosmological problem, some multi-brane extensions of the RS model (still addressing the gauge hierarchy problem), in which the SM fields are stuck on a positive tension brane, have been constructed [13–17]. In the RS model extension of [13, 14], two positive tension branes are sitting on the fixed points of the $S^1/\mathbb{Z}_2$ orbifold, namely at $y = 0$ and $y = \pm\pi R_c$, and a third parallel 3-brane with negative tension can move freely in between. Within this "$+ - +$" scenario, our universe is the "+" brane at $y = \pm\pi R_c$ (the two brane RS model is denoted as "$+-$" accordingly to this terminology). In another possible RS extension: the "$+ + -$" model [13, 15], a "+" brane is located at $y = 0$, a "$-$" brane at $y = \pm\pi R_c$ and we live on an intermediate parallel "+" brane. In the "++" model [16, 17], two "+" branes are sitting on the two orbifold fixed points and we live on one of them.

It must be first mentioned that the original motivation for the multi-brane RS extensions (which is to provide a solution to the cosmological problem, arising in the RS model, related to the modification of the Friedmann equations) is not as strong as it seems. Indeed, in the presence of a mechanism stabilizing the size of the extra dimension within the RS model (like the mechanisms suggested in [32] or [33]), the ordinary FRW (Friedmann–Robertson–Walker) equations are recovered [34]. However, the multi-brane RS extensions remain interesting alternatives to the initial version of RS model since they give rise to a specific phenomenology. As a matter of fact, the multi-brane RS extensions possess a specific feature: the first KK excitations of a bulk fermion (or the graviton) are typically anomalously light (relatively to the electroweak scale) [35, 36][1]. The reason being that the magnitude of wave functions for the first fermion KK modes typically approximates closely that for the 0-mode, differing significantly only in a region where the 0-mode wave function is exponentially suppressed.

Several other models, which extend the original RS framework, have also been elaborated in the literature, including the possibilities of configurations with several branes [38–40], intersecting branes [41, 42] or non-compact extra dimension(s) [43–46] (as for instance in the case of an infinite crystalline universe [15, 47–49]). Those theoretical models lead generally to a rather complicated phenomenology.

Nevertheless, within all these attractive scenarios, namely ADD, RS and its multi-brane extensions, understanding the lightness of neutrinos (with respect to the electroweak energy scale) becomes a challenge. Indeed, in this new class of brane universe models, the (effective) mass scale of gravity is of the order of TeV so that there exist no high energy scales. Hence, this brane world picture conflicts with the traditional interpretation of small neutrino mass size invoking the "see-saw" mechanism [50–52], which requires a superheavy mass scale close to the GUT scale ($\sim 10^{16}$ GeV).

In the context of the ADD scenario, a novel type of explanation for the smallness of neutrino masses has been proposed in terms of purely geometrical means [53, 54] (the associated neutrino phenomenology has been extensively studied in [55–73, 75])[2]. We recall here briefly the basic ideas of this kind of explanation, which does not rely on the existence of any high energy scale. The starting point is the observation that a right-handed neutrino added to the SM would be a gauge singlet, and could thus propagate freely inside the bulk. In such a situation, small Dirac neutrino masses can be naturally generated as the Yukawa couplings, between the Higgs boson, SM left-handed neutrinos and zero mode of bulk right-handed neutrinos, are suppressed, due to the weak interaction probability of bulk neutrinos with SM fields (which are confined to our 4-dimensional subspace). This suppression is considerable since, in the ADD framework, the volume of extra compact space is large relative to the thickness of the wall where SM states propagate. More precisely, the effective 4-dimensional mass term, between SM neutrinos and zero mode of bulk neutrinos, involves a suppression factor of the form (see (1))

$$m_\nu = \kappa\ (M^n V_n)^{-1/2}\ \upsilon = \kappa\ (M/M_{\rm Pl})\ \upsilon, \qquad (4)$$

where κ represents the dimensionless Yukawa coupling constants and $\upsilon \simeq 174\,\text{GeV}$ the vacuum expectation value (VEV) of the SM Higgs boson. The physical neutrino masses are the eigenvalues of the whole neutrino mass matrix, which involves the masses of type (4), but also the masses of the Kaluza–Klein (KK) excitations of bulk right-handed neutrinos, as well as the masses mixing those KK modes with the SM left-handed neutrinos (which originate from the Yukawa couplings).

The same kind of intrinsically higher-dimensional mechanism as above (with an additional right-handed neutrino in the bulk), producing small neutrino masses, can apply to

[1] In contrast, within the pure RS framework, the first KK modes of bulk fermions have typically electroweak-scale masses [37].

[2] Let us mention that, within the ADD framework, other types of higher-dimensional mechanism, which might permit the generation of small neutrino masses, have been suggested: the lightness of neutrinos could result from the power-law running of Yukawa couplings [53] or the breaking of lepton number on distant branes in the bulk [54, 76].

the RS model case[3]. Nevertheless, within the RS scenario, the compactification volume is small ($2\pi R_c \sim 70 M_{\rm Pl}^{-1}$) and the AdS_5 geometry tends to localize the 0-mode of bulk neutrino around the brane where are stuck SM particles (TeV-brane)[4]. Hence, in order to obtain the desired order of magnitude for the effective neutrino mass scale, via this higher-dimensional mechanism, one has to introduce a 5-dimensional mass term for the bulk right-handed neutrino [37] (see also [79] for the specific case of a 7-dimensional spacetime including one warped extra dimension). Indeed, such a mass term of the appropriate form can modify the wave function for the 0-mode[5] of bulk neutrino (applying the ideas of [4, 84]) so that its overlap with the TeV-brane, and thus its effective 4-dimensional Yukawa coupling with the Higgs boson and SM left-handed neutrino, appears to be greatly reduced.

We mention here an alternative possibility, although it corresponds to a different physical context from the one considered in the present paper: in the RS model, all SM fields (except the Higgs boson, otherwise the gauge hierarchy problem would reappear [85, 86]) could live inside the bulk [85, 87, 88], if the SM gauge group is enhanced (in order to satisfy the electroweak precision constraints [89]). This realistic hypothesis provides a new way for interpreting the flavor structure of SM fermion masses [88, 90–92].

In the present work, we address the question which arises naturally from the above discussion: does the same type of geometrical mechanism as above (with a 5-dimensional mass term for the bulk right-handed neutrino) enable one to create reasonable neutrino mass values in multi-brane extensions of the RS scenario?

One expects that indeed sufficiently reduced neutrino masses can again be achieved through this mechanism. However, within the multi-brane RS extensions, the first KK excitations of right-handed neutrino can acquire ultra-light masses compared to the electroweak scale (see above). By consequence, some mixing (induced by the Yukawa couplings) angles between the SM left-handed neutrinos and KK excitations of right-handed neutrinos should be typically large. Now, one can obtain severe experimental upper bounds on values of this kind of mixing angle between SM neutrinos and KK modes of right-handed neutrinos, since those KK states constitute new sterile neutrinos with respect to gauge interactions. In particular, these mixing angles can be strongly constrained by considering the experimental data on Z^0 boson widths associated to certain decay channels.

Therefore, we will derive these constraints on mixing angles (arising due to the presence of bulk right-handed neutrinos) which are issued from the measurements of Z^0 boson widths, within the context of multi-brane generalizations of the RS model. Then, we will determine whether those constraints translate into bounds, on the theoretical parameters, which do or do not exclude multi-brane extensions of the RS model from the possible frameworks addressing simultaneously the gauge hierarchy and small neutrino mass questions (the other experimental constraints on neutrino mass matrix will also be considered).

The organization of this paper is as follows. In Sect. 2, we describe in details the entire neutrino mass matrix, within the "$+-+$" framework, in case additional right-handed neutrinos propagate in the bulk. Then, in Sect. 3, by considering this neutrino mass matrix, we derive and discuss constraints on the "$+-+$" scenario coming from experimental bounds which concern neutrino physics. Based on our understanding of the "$+-+$" analysis performed in those two sections, we discuss in Sect. 4 the cases of the other multi-brane RS extensions: the "$++$" and "$++-$" models. Finally, we conclude in Sect. 5.

2 Neutrino masses in the "$+-+$" model

2.1 The "$+-+$" model

The goal of this work is a phenomenological study on the new paradigm of anomalously light KK excitations which is associated to the multi-brane RS extensions. We will begin by concentrating on a certain concrete realization, namely the "$+-+$" model, of this new paradigm because of the relative simplicity of calculations within the "$+-+$" context.

We mention here that the "$+-+$" model[6] can be seen as a limiting case of the more general "$+--+$" brane universe scenario (with a flat space between the two "$-$" branes) [15, 93].

The "$+-+$" model suffers from the presence of unacceptable radion fields, associated with the perturbations of the freely moving "$-$" brane sandwiched between "$+$" branes, which are necessarily ghost states with negative kinetic term [94, 95]. This fact is indeed problematic regarding the construction because the system is probably quantum mechanically unstable. Classically, the origin of the problem is connected to the violation of the weaker energy condition [96, 97].

However, we will consider the "$+-+$" model as a "toy model" and a calculation tool allowing us then to develop a

[3] Within the RS context, small (Dirac) neutrino masses can also be generated by another kind of model [77], in which the lepton number symmetry is explicitly broken on the Planck brane while the right-handed neutrino is localized on the TeV-brane. Remarkably, because of the AdS/CFT correspondence, there exists a purely 4-dimensional dual description of such models, where the right-handed neutrino is a composite bound state (composite right-handed neutrinos were independently studied in [78]).

[4] Unlike the 0-mode of graviton which is localized on the positive tension branes, the 0-mode of bulk fermions are localized on the negative tension ones (see (2)).

[5] In the RS model, bulk fermions possess systematically a 0-mode [37], in contrast with the 0-mode of bulk scalar [80] and vector [81–83] fields which exists only for vanishing mass in the fundamental theory.

[6] The regions of parameter space that we consider in the present work, namely the regions where the gauge hierarchy problem can effectively be solved, do not correspond to the exotic Bi-gravity limit (giving rise to modifications of gravity at extremely large distances [13]) of the studied "+-+" model.

better understanding of the phenomenological analyzes on the other multi-brane RS extensions, namely the "++" and "$++-$" models (which will be treated later; see Sect. 4). Now, those "++" and "$++-$" models represent concrete realizations of the mentioned new paradigm which do not suffer from the presence of radion ghost states.

Moreover, a 6-dimensional version, totally consistent from the theoretical point of view, of the "$+-+$" model has been elaborated [98]. Within this 6-dimensional framework, due to the non-trivial internal space, the characteristic bounce of warp factor can appear even without the presence of any (moving) "$-$" brane. Hence, one can avoid here the problematic presence of radion ghost states.

Now, in this 6-dimensional setup, the "+" 3-branes of the "$+-+$" model are replaced by 4-branes, but one of their dimensions is compact, unwarped and of Planck length. Thus, in the low energy limit the spacetime on the branes appears 3-dimensional. The 6-dimensional "$+-+$" version has similar predictions and almost identical properties as the considered 5-dimensional "+-+" version, so that one can extend the present phenomenological study to the case of a 6-dimensional spacetime.

Besides, there exists a potential way out of the radion ghost problem arising in the "$+-+$" model: the "$-$" brane can be replaced by an external constant 4-form field which would mimic the effects of such a brane [99,100].

One can also assume that there exist a specific framework in which the radion ghost fields, appearing in the "$+-+$" model, condensate (so that the background can be stabilized) [101] or that the theory containing ghosts may in fact be viable due to other kinds of particular circumstances [102]. An alternative solution would be that there exist a certain regularization procedure (like the one proposed in the recent works [103, 104]) which applies to the "$+-+$" model, resulting in a theory free from ghosts or tachyons.

2.2 Formalism of the "$+-+$" model

In view of a comparison with the "$+-+$" scenario, we recall that within the RS model, the considered solution to the 5-dimensional Einstein's equations, respecting 4-dimensional Poincaré invariance, leads to the non-factorisable metric:

$$\mathrm{d}s^2 = \mathrm{e}^{-2\sigma(y)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}y^2, \qquad (5)$$

with $\sigma(y) = k|y|$, x^μ $[\mu = 1, \dots, 4]$ the coordinates for the familiar 4 dimensions and $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ the 4-dimensional metric. The bulk geometry, associated to the metric (5), is a slice of AdS_5 space. By considering the fluctuations of metric (5), one obtains, after integration over y, the expression for the effective 4-dimensional Planck scale M_{Pl}:

$$M_{\mathrm{Pl}}^2 = \frac{M_5^3}{k}\left(1 - \mathrm{e}^{-2\pi k R_\mathrm{c}}\right). \qquad (6)$$

Within the "$+-+$" model, the tensions of the "+" brane sitting at $y = 0$, the "$-$" brane at $y = L_-$ ($L_- > 0$) and the "+" brane at $y = L_+ = \pi R_\mathrm{c}$ ($L_+ > L_-$) are tuned to (to be compared with (2))

$$\Lambda_{(y=0)} = -\Lambda_{(y=L_-)} = \Lambda_{(y=L_+)} = -\Lambda/k = 24kM_5^3. \quad (7)$$

In this framework, the metric ansatz, that should respect the 4-dimensional Poincaré invariance, is taken as in (5) with the following solution for the function $\sigma(y)$:

$$\sigma(y) = k(L_- - ||y| - L_-|). \qquad (8)$$

This solution can only be trusted for an AdS_5 curvature smaller than the fundamental 5-dimensional Planck scale:

$$k < M_5. \qquad (9)$$

In the "$+-+$" extension of the RS model, the expression for the effective 4-dimensional Planck scale M_{Pl} becomes (to be compared with (6))

$$M_{\mathrm{Pl}}^2 = \frac{M_5^3}{k}\left(1 - 2\mathrm{e}^{-2kL_-} + \mathrm{e}^{-2k(2L_- - L_+)}\right). \qquad (10)$$

Besides, for the solution (8), the warp factor defined by (3), in which $M_\star$ denotes now the effective Planck mass on the "+" brane at $y = L_+ = \pi R_\mathrm{c}$ (where are confined all SM fields), reads

$$w = \mathrm{e}^{-\sigma(L_+)} = \mathrm{e}^{-k(2L_- - L_+)}. \qquad (11)$$

Hence, in the "$+-+$" scenario, the gauge hierarchy problem is solved for $M_\star = \mathcal{O}(\mathrm{TeV})$ which is achieved when the warp factor verifies

$$w \sim 10^{-15}, \qquad (12)$$

or equivalently:

$$2L_- - L_+ \sim 34/k. \qquad (13)$$

In view of future discussions, we introduce the quantity x defined by

$$x = k(L_+ - L_-). \qquad (14)$$

Then (10) can be rewritten in terms of the parameters w and x:

$$M_5^3 = kM_{\mathrm{Pl}}^2\left[1 + w^2\left(1 - 2\mathrm{e}^{-2x}\right)\right]. \qquad (15)$$

2.3 Neutrino mass matrix

In this section, we describe all the relevant contributions to the neutrino mass matrix. The higher-dimensional mechanism generating Dirac neutrino masses, that we study in this paper, requires a 5-dimensional mass term for the additional bulk neutrino (see Sect. 1). In the "$+-+$" background considered here, this mass term enters the 5-dimensional action of bulk neutrino as

$$\mathcal{S}_5 = \int \mathrm{d}^4x \int \mathrm{d}y\, \sqrt{G}$$
$$\times \left(E_a^M \left[\frac{\mathrm{i}}{2}\bar{\Psi}\gamma^a \left(\overrightarrow{\partial_M} - \overleftarrow{\partial_M}\right)\Psi + \frac{\omega_{bcM}}{8}\bar{\Psi}\left\{\gamma^a, \sigma^{bc}\right\}\Psi \right]\right.$$

$$-m\bar{\Psi}\Psi - \lambda_5 H\bar{L}\Psi + \text{h.c.}\Big), \quad (16)$$

$G = \det(G_{MN}) = \mathrm{e}^{-8\sigma(y)}$ (with $\sigma(y)$ as given in (8))[7] being the determinant of the metric, $E^M_a = \mathrm{diag}(\mathrm{e}^{\sigma(y)}, \mathrm{e}^{\sigma(y)}, \mathrm{e}^{\sigma(y)}, \mathrm{e}^{\sigma(y)}, 1)$ the inverse vielbein, $\Psi = \Psi(x^\mu, y)$ the neutrino spinor, $\gamma^a = (\gamma^\mu, \mathrm{i}\gamma_5)$ the 4-dimensional representation of Dirac matrices in 5-dimensional flat space, ω_{bcM} the spin connection (c.f. [35]), m the neutrino mass in the fundamental theory, λ_5 the Yukawa coupling constant (of mass dimension $-1/2$), H the Higgs boson field and L the SM lepton doublet.

In order to localize the 0-mode of bulk neutrino, the mass m must have a non-trivial dependence on the fifth dimension, and more precisely with a '(multi-)kink' profile [4,84]. The mass m could be the VEV of a scalar field. We consider the economic possibility that this scalar field has a double rôle, in the sense that it also creates the branes themselves [105] which imposes the following condition on the VEV:

$$m = c\,\frac{\mathrm{d}\sigma(y)}{\mathrm{d}y}, \quad (17)$$

where c is a dimensionless parameter and $\sigma(y)$ is defined by (8). We check that the VEV (17) is well compatible with the $\mathbb{Z}_2$ symmetry ($y \to -y$) of the $S^1/\mathbb{Z}_2$ orbifold: this VEV is odd under the $\mathbb{Z}_2$ transformation (see (8)), like the product $\bar{\Psi}\Psi$ (as fermion parity is defined by $\Psi(-y) = \pm\gamma_5\Psi(y)$), so that the term $m\bar{\Psi}\Psi$ is even which allows one to preserve the invariance of action (16).

At this stage, what do we know about the value of parameter c? Since the mass m is a parameter that appears in the original 5-dimensional action (16), its natural absolute value is of the order of the fundamental 5-dimensional Planck scale M_5. Besides, $\mathrm{d}\sigma(y)/\mathrm{d}y = \pm k$ (see (8)) and $k < M_5$ (see (9)). Hence, it is clear from (17) that the "physical" value of c verifies: $c > 1$ (as discussed in [35]).

By consequence, the relevant case is the one characterized by $c > 1/2$, in which the 0-mode of bulk neutrino is localized on the two positive tension branes (at $y = 0$ and $y = L_+$) [36] like the 0-mode of graviton (as mentioned in footnote 4). Therefore, the effective 4-dimensional mass $m^{(0)}_\nu$, induced by the Yukawa coupling of action (16) and mixing the 0-mode of bulk right-handed neutrino $\psi^{(0)}_{\mathrm{R}}$ with the SM left-handed neutrino ν_{L} (stuck on the "+" brane at $y = L_+$), is reduced for the same geometrical reason that the effective scale of gravity $M_\star$ (on the brane sitting at $y = L_+$) is suppressed. This mass $m^{(0)}_\nu$ can thus be expressed (for $c > 1/2$) in term of the warp factor w defined by (3) and (11) [35]:

$$m^{(0)}_\nu \simeq \sqrt{\frac{k}{M_5}\left(c - \frac{1}{2}\right)}\; w^{c-1/2}\; \upsilon. \quad (18)$$

In this expression, the 5-dimensional Yukawa parameter λ_5 has been taken to have its natural value: $\lambda_5 \simeq M_5^{-1/2}$.

[7] We use the capital indexes M, N, … for objects defined in 5-dimensional curved space, and the lower-case indexes a, b, … for objects defined in the tangent frame.

Similarly, the Yukawa coupling of action (16) also induces the following effective 4-dimensional masses mixing ν_{L} with the first KK excitation of neutrino $\psi^{(1)}_{\mathrm{R}}$ [35]:

$$m^{(1)}_\nu \simeq \sqrt{\frac{k}{M_5}\left(c - \frac{1}{2}\right)}\; \upsilon, \quad (19)$$

or with the other KK excitations $\psi^{(n)}_{\mathrm{R}}$ $[n > 1]$ [35]:

$$m^{(n)}_\nu \simeq \sqrt{\frac{k}{M_5}\left(c - \frac{1}{2}\right)}\; \mathrm{e}^{-x}\; \upsilon \qquad [n = 2, 3, 4, \ldots]. \quad (20)$$

The x dependence in (20) can be understood as follows: the KK states $\psi^{(n)}_{\mathrm{R}}$ $[n > 1]$ are localized around the "−" brane at $y = L_-$ (in contrast with the first KK mode $\psi^{(1)}_{\mathrm{R}}$ which is localized on the two positive tension branes) so that when x increases their wave functions overlap with ν_{L} (trapped on the "+" brane at $y = L_+$), and thus the associated mass $m^{(n)}_\nu$ decreases.

Finally, the excited modes of bulk neutrino $\psi^{(n)}$ $[n \geq 1]$ acquire KK masses of the form (for $c > 1/2$) [36]:

$$m^{(1)}_{\mathrm{KK}} = \sqrt{4c^2 - 1}\; w\; \mathrm{e}^{-(c+1/2)x}\; k, \quad (21)$$

$$m^{(n+1)}_{\mathrm{KK}} = \xi_n\; w\; \mathrm{e}^{-x}\; k \qquad [n = 1, 2, 3, \ldots], \quad (22)$$

where ξ_{2i+1} is the $(i+1)$th root of $J_{c-1/2}(X) = 0$ $(i = 0, 1, 2, \ldots)$ and ξ_{2i} is the ith root of $J_{c+1/2}(X) = 0$ $(i = 1, 2, 3, \ldots)$, $J_{c\pm 1/2}(X)$ denoting the Bessel functions of the first kind and order $c \pm 1/2$. We remark that the KK mass $m^{(1)}_{\mathrm{KK}}$ of the first excited state $\psi^{(1)}$ is manifestly singled out from the rest of the KK tower.

In conclusion, within the framework we study (namely the "+ − +" scenario with an additional massive bulk neutrino), the complete neutrino mass matrix appears in the effective lagrangian as

$$\mathcal{L} = -\bar{\psi}^\nu_{\mathrm{L}} \mathcal{M} \psi^\nu_{\mathrm{R}} + \text{h.c.}, \quad (23)$$

$\psi^\nu_{\mathrm{L,R}}$ representing the 4-dimensional fields: $\psi^\nu_{\mathrm{L}} = (\nu_{\mathrm{L}}, \psi^{(1)}_{\mathrm{L}}, \psi^{(2)}_{\mathrm{L}}, \ldots)$ and $\psi^\nu_{\mathrm{R}} = (\psi^{(0)}_{\mathrm{R}}, \psi^{(1)}_{\mathrm{R}}, \psi^{(2)}_{\mathrm{R}}, \ldots)$, and reads (see (18)–(22))

$$\mathcal{M} = \begin{pmatrix} m^{(0)}_\nu & m^{(1)}_\nu & m^{(2)}_\nu & \ldots \\ 0 & m^{(1)}_{\mathrm{KK}} & 0 & \ldots \\ 0 & 0 & m^{(2)}_{\mathrm{KK}} & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (24)$$

3 Experimental constraints on the "+ − +" model from neutrino physics

3.1 Number of neutrino generations

3.1.1 Bound from Z^0 width measurements

If the SM left-handed neutrino mixes with some sterile (with respect to the SM gauge interactions) left-handed neutrinos, then its effective weak charge is diminished [91]. That is the reason why, in such a situation, the measurements of Z^0 boson width induce a constraint on the mixing angles between sterile left-handed neutrinos and the SM one. In our framework, we must study this constraint since the SM neutrino $\nu_{\rm L}$ mixes with the KK excitations of bulk neutrino $\psi_{\rm L}^{(n)}$ $[n \geq 1]$ (see Sect. 2.3), which constitute sterile left-handed neutrinos.

Here, we derive this constraint in the considered physical context. Let us define the quantity n_ν as

$$n_\nu = \frac{\Gamma^{\rm exp}(Z^0 \to \text{invisible})}{\Gamma^{\rm th}(Z^0 \to \bar{\nu}_{\rm L}\nu_{\rm L})}, \tag{25}$$

where $\Gamma^{\rm exp}(Z^0 \to \text{invisible})$ stands for the experimental data on the Z^0 boson width associated to the decay channel into any undetectable particle, and $\Gamma^{\rm th}(Z^0 \to \bar{\nu}_{\rm L}\nu_{\rm L})$ represents the known theoretical prediction of the Z^0 boson width associated to the decay into a single family of SM neutrino (neutrino masses being neglected relative to the Z^0 mass). The value of n_ν obtained experimentally is [106]

$$n_\nu = 2.985 \pm 0.008. \tag{26}$$

In the absence of any sterile neutrino effect, n_ν is nothing but an experimental estimation of the number of SM neutrino generations. Since in our framework, the SM neutrinos mix not only with each other but also with the sterile neutrinos $\psi_{\rm L}^{(n)}$, their effective weak coupling is suppressed so that the number of SM neutrino generations reads [37]

$$n_\nu^{\rm gen} = 3 = n_\nu / \cos^2\theta_\nu, \tag{27}$$

where $\cos^2\theta_\nu$ represents the admixture of lightest neutrino eigenstates for the SM electron neutrino, in case this admixture is identical for the muon and tau neutrinos. The lightest neutrino eigenstates stand here for all the neutrino eigenstates with a mass smaller than half the Z^0 boson mass (so that they can be produced in the Z^0 decay). The quantity $\cos^2\theta_\nu$ involves thus mixing angles between SM left-handed neutrinos and sterile neutrinos $\psi_{\rm L}^{(n)}$. Equations (26) and (27) lead to the (expected) bound on this angle θ_ν:

$$\tan^2\theta_\nu < 0.0077. \tag{28}$$

What is the precise definition of $\cos\theta_\nu$ in the case of a unique lepton flavor (the three flavor case will be treated in Sect. 3.3.4)? This definition is

$$\cos\theta_\nu = U_{01}, \tag{29}$$

where U is the unitary matrix responsible for the basis transformation:

$$\psi_{\rm L}^\nu = U\ \psi_{\rm L}^{\rm phys}, \tag{30}$$

$\psi_{\rm L}^{\rm phys}$ containing the neutrino mass eigenstates: $\psi_{\rm L}^{\rm phys} = (\nu_1, \nu_2, \nu_3, \ldots)$ (and $\psi_{\rm L}^\nu$ being defined in Sect. (2.3)). The two indexes 0 and 1 of matrix element (29) correspond respectively to the SM neutrino $\nu_{\rm L}$ in the vector $\psi_{\rm L}^\nu$ and to the lightest neutrino eigenstate ν_1 in vector $\psi_{\rm L}^{\rm phys}$. More explicitly, one has $\nu_{\rm L} = U_{01}\ \nu_1 + \ldots$ Here, we have assumed that only the lightest neutrino eigenstate ν_1 has a mass smaller than half the Z^0 mass (The hypothesis that more neutrino eigenstates have masses smaller than half the Z^0 mass will be discussed in Sect. 3.3.3).

3.1.2 Implications for the parameters of the "$+-+$" model

In this part, we will translate the experimental bound (28) into a constraint on the fundamental parameters in the version of the "$+-+$" scenario with a massive bulk right-handed neutrino.

The mixing angle θ_ν entering (28), and defined in (29)–(30), is calculated in Appendix B, in case the neutrino mass matrix is given by (24) which is characteristic of the presence of a bulk right-handed neutrino (eigenvalues of the hermitian square of matrix (24) are discussed in Appendix A). The result appears in (B.7). The KK masses and masses mixing the SM neutrino with excited states of bulk neutrino, which enter (B.7), can be replaced by their expression within the "$+-+$" framework given in (21)–(22) and (19)–(20) respectively: this leads to the following expression for $\tan^2\theta_\nu$ in terms of the fundamental parameters:

$$\tan^2\theta_\nu \simeq \frac{v^2}{w^2 k M_5} \times \left[\frac{\mathrm{e}^{(2c+1)x}}{2(2c+1)} + \left(c - \frac{1}{2}\right)\left(\sum_{i=1}^{\infty} \frac{1}{(\zeta_i^+)^2} + \sum_{i=1}^{\infty} \frac{1}{(\zeta_i^-)^2}\right)\right], \tag{31}$$

where ζ_i^+ and ζ_i^- are the ith roots ($i = 1, 2, 3, \ldots$) of $J_{c+1/2}(X) = 0$ and $J_{c-1/2}(X) = 0$ respectively. The two infinite sums of (31) can be performed exactly and yield

$$\tan^2\theta_\nu \simeq \frac{v^2}{w^2 k M_5}\left[\frac{\mathrm{e}^{(2c+1)x}}{2(2c+1)} + g(c)\right],$$

with

$$g(c) = \frac{(c+1)(2c-1)}{(2c+3)(2c+1)}. \tag{32}$$

From this expression of $\tan^2\theta_\nu$ and the experimental bound (28) on $\tan^2\theta_\nu$, we deduce the following constraint on the theoretical parameters x, w, k, M_5 and c (to be added to the SM parameters) of the "$+-+$" model with an additional massive bulk neutrino:

$$x \lesssim \frac{1}{2c+1} \ln\left[0.0077 \times 2(2c+1)\frac{w^2 k M_5}{v^2} - \frac{(2c-1)(2c+2)}{(2c+3)}\right]. \tag{33}$$

The ln function involved in (33) is well defined on the intervals of parameters that we will consider (see (41)).

It is instructive to remark that, in fact, the upper bound (28) on $\tan^2\theta_\nu$ has been expressed as an upper bound on the parameter x (c.f. (33)). This point can be understood in a physical way as follows. The dominant effect of a decrease of x on the matrix (24) is that the KK masses (21)–(22) for excited modes of bulk neutrino increase, so that these excited modes $\psi^{(n)}$ $[n \geq 1]$ tend to decouple. This induces a decrease of the mixing, quantified typically by $\tan^2\theta_\nu$ (see Sect. 3.1.1), between the left-handed component $\psi_{\rm L}^{(n)}$ of those sterile neutrinos $\psi^{(n)}$ $[n \geq 1]$ and the SM neutrino $\nu_{\rm L}$.

3.2 Neutrino masses

In this section, we will determine the constraints on the parameters of the "$+-+$" scenario, with an additional massive bulk neutrino, originating from the experimental bounds on neutrino masses. We will concentrate on the experimental bounds on absolute neutrino mass scales: here, the relevant bounds are those extracted from the tritium beta decay experiments [107–110] since those bounds are independent of whether neutrinos are Majorana or Dirac particles. In contrast, the other bounds issued from neutrinoless double beta decay experiments (see [111] for a review) apply only on Majorana neutrino masses and thus do not hold in the present framework where neutrinos acquire Dirac masses (see (23)).

The best limit coming from data on tritium beta decay has been obtained by the Mainz experiment and reads [108]

$$m_\beta \leq 2.2\,\text{eV} \quad (95\%\ \text{C.L.}). \tag{34}$$

We also indicate the limit extracted from data on tritium beta decay measured by the Troitsk experiment: $m_\beta \leq 2.5$ eV (95% C.L.) [109]. In the assumption of no mixing between lepton flavors, the quantity $m_\beta = m(\nu_e)$ introduced in (34) is the electron neutrino mass, or equivalently the associated neutrino mass eigenvalue [107, 110]. Therefore, in our physical context with only one lepton flavor (the electron flavor), the experimental limit (34) can be applied on the smallest neutrino mass eigenvalue m_{ν_1} (see end of Sect. 3.1.1 and (A.1)) which leads then to

$$m_{\nu_1} \leq 2.2\,\text{eV}. \tag{35}$$

It must be mentioned that under the realistic hypothesis of three mixing lepton flavors, the effective mass, to which are sensitive the tritium beta decay experiments, reads $m_\beta = \left(\sum_{i=1}^3 |U_{ei}^{MNS}|^2\ m_{\nu_i}^2\right)^{1/2}$ where U^{MNS} is the lepton mixing matrix [107, 110]. In this case, taking into account the experimental lepton mixing angle values [112] and small squared neutrino mass differences ($|m_{\nu_2}^2 - m_{\nu_1}^2| \in [6.1, 8.4] \times 10^{-5}\,\text{eV}^2$ and $|m_{\nu_3}^2 - m_{\nu_1}^2| \in [1.4, 3.0] \times 10^{-3}\,\text{eV}^2$ at 2σ from a global data analysis [112][8]) obtained from oscillation experiments, one would expect that (34) still leads to upper bounds on the three weakest neutrino mass eigenvalues $m_{\nu_{1,2,3}}$ of the order of eV (as in (35)).

Besides, in the case of three lepton flavors, the upper cosmological bound $\sum_{i=1}^3 m_{\nu_i} < 0.7 \leftrightarrow 1.01\,\text{eV}$ (depending on cosmological priors), which comes from WMAP and 2dFGRS galaxy survey [113], also corresponds to limits on the three weakest neutrino mass eigenvalues $m_{\nu_{1,2,3}}$ of the same order as in (35).

An expression for the smallest neutrino mass eigenvalue m_{ν_1}, which enters (35), can be found by combining (A.8) and (B.7). The result is

$$m_{\nu_1} \simeq \cos\theta_\nu\ m_\nu^{(0)}. \tag{36}$$

The expression (36), together with (18) and (35), leads to the following experimental constraint on fundamental parameters:

$$\cos\theta_\nu \sqrt{\frac{k}{M_5}\left(c - \frac{1}{2}\right)}\ w^{c-1/2}\ \upsilon \lesssim 2.2\,\text{eV}. \tag{37}$$

3.3 Combination of the constraints

3.3.1 Constraint on the parameter c

Here, we deduce from (37) (representing an experimental bound on neutrino mass) a constraint on theoretical parameter c (defined by (17)). For the value of ratio k/M_5 involved in (37), we consider the range $10^{-4} \lesssim k/M_5 \lesssim 1$, its upper boundary being motivated by (9) and its lower one by the fact that it is not desirable to introduce a new high hierarchy between the AdS_5 curvature k and the fundamental scale of gravity M_5. Hence, by using the value of warp factor given in (12) (for which the gauge hierarchy problem is solved), the bound (28) and (37), we obtain the numerical results:

$$c \gtrsim 1.08 \quad \text{for } k/M_5 = 10^{-4}, \tag{38}$$

$$c \gtrsim 1.22 \quad \text{for } k/M_5 = 1. \tag{39}$$

Those results mean that the obtained value of the lower limit on parameter c lies typically in the interval $[1.08, 1.22]$ if $10^{-4} \lesssim k/M_5 \lesssim 1$. The dependence of this lower limit for c on the warp factor value is weak: for instance, if $w = \{10^{-13}; 10^{-14}; 10^{-15}; 10^{-16}; 10^{-17}\}$ then (39) reads $c \gtrsim \{1.33; 1.27; 1.22; 1.17; 1.13\}$ $[k/M_5 = 1]$ respectively.

3.3.2 Constraint on the parameter k

In fact, the bound (33) on the parameter x allows one to impose a constraint on the AdS_5 curvature k. Let us derive this constraint. For that purpose, we first observe that, within the considered "$+-+$" model, the quantity x defined by (14) is positive (see Sect. 2.2), namely

$$x > 0. \tag{40}$$

Indeed, the opposite case $x < 0$ ($\Leftrightarrow 0 < L_+ < L_-$) corresponds to the brane configuration of the "$++-$" scenario, in which a "$+$" brane (at $y = L_+$) is sitting between another "$+$" brane (at $y = 0$) and a "$-$" brane (at $y = L_-$). (33) and (40) lead to

$$0.0077 \times 2(2c+1)\frac{w^2 k M_5}{\upsilon^2} - \frac{(2c-1)(2c+2)}{(2c+3)} \gtrsim 1. \tag{41}$$

[8] The study performed in [112] is based on the results of the atmospheric and solar neutrino experiments as well as the accelerator (K2K) and reactor (CHOOZ and KamLAND) experiments.

By considering (41) together with the limit (38), which is conservative all through the considered range $10^{-4} \lesssim k/M_5 \lesssim 1$ (see Sect. 3.3.1), we find the following numerical result:

$$w\sqrt{kM_5} \gtrsim 1.1\,\mathrm{TeV}. \qquad (42)$$

Now, by taking into account the relevant values of the brane positions L_- and L_+ (see (47), presented later), the relation (10) (characteristic of the "$+-+$" model) can be rewritten in a good approximation as

$$M_5^3 \simeq kM_{\mathrm{Pl}}^2. \qquad (43)$$

This new relation, the condition (9) and the value of warp factor (12) (required in order to solve the gauge hierarchy problem) can be combined to give $w\sqrt{kM_5} \leq \mathcal{O}(\mathrm{TeV})$, which leads, together with (42), to the important result

$$w\sqrt{kM_5} = \mathcal{O}(\mathrm{TeV}). \qquad (44)$$

This result and the M_5 expression (43) give rise to the following expected constraint on curvature parameter k (for the value of w given by (12)):

$$k \sim M_5 \sim M_{\mathrm{Pl}}. \qquad (45)$$

Let us make an important comment on the relation found in (45): this relation, $k \sim M_5$, can be interpreted as a condition of "naturality" (fixing the AdS_5 curvature parameter k) within the generic "$+-+$" framework. In the sense that this relation avoids the possibility to introduce a new strong hierarchy between the energy scale k and fundamental Planck scale M_5 in the "$+-+$" model (the main interest of the "$+-+$" model being to solve the problem of strong hierarchy between the electroweak scale and scale M_5).

3.3.3 Constraint on the parameter x

(1) Numerical values: Let us present and discuss the values for limit (33) on the x parameter of the "$+-+$" model. We recall that this limit is nothing else but an expression of the experimental bound (28) originating from considerations on the number of neutrino families.

In Fig. 1, we show the value of this limit (33) on x as a function of the parameter c. The other quantity $w\sqrt{kM_5}$, on which also depends the limit (33), has been set around the TeV scale in this figure. This choice is motivated by (44) which results from a combination of various constraints.

The behavior of curves drawn on Fig. 1 can be explained physically in the following terms. The decrease of c has two dominant effects on matrix (24). The first one is a decrease of the masses (19)–(20) mixing the SM neutrino ν_{L} with KK excitations of bulk neutrino $\psi^{(n)}$ $[n \geq 1]$. The second one is that the KK mass (21) for the first excited mode $\psi^{(1)}$ increases, so that this excited state $\psi^{(1)}$ tends to decouple. These two effects induce a decrease of the mixing, quantified typically by $\tan^2\theta_\nu$ (see Sect. 3.1.1), between the SM neutrino ν_{L} and sterile neutrinos $\psi^{(n)}$ $[n \geq 1]$. Therefore, a c decrease can be compensated by an

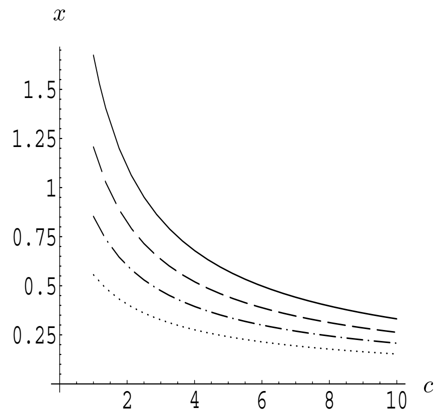

Fig. 1. Value of the upper bound on x obtained in (33) as a function of parameter c, for $w\sqrt{kM_5}$ equal to 2 TeV (dotted line), 3 TeV (dot-dashed line), 5 TeV (dashed line) and 10 TeV (plain line). The choice of those values for the parameter combination $w\sqrt{kM_5}$ is motivated by (42) and (44). The regions situated above the curves are rejected by bound (33)

increase of x in a way such that $\tan^2\theta_\nu$ remains fixed at a given value, since $\tan^2\theta_\nu$ increases with x (see (32)) as we have explained at the end of Sect. 3.1.2. This feature allows one to understand why in Fig. 1 the x value, which is associated to a value of $\tan^2\theta_\nu$ fixed to its limit: 0.0077 (as the x bound represented in Fig. 1 expresses the bound (28) on $\tan^2\theta_\nu$), increases when c diminishes, all other fundamental parameters (w, k and M_5) being fixed.

(2) Bound on x from the combination of constraints on c, k and x: Motivated by the constraint on k obtained in (45) and the condition (12) concerning gauge hierarchy, we choose to consider (39) as the relevant constraint on c originating from experimental bounds on neutrino masses. As it is clear from Fig. 1, by combining this constraint (39) on c with the constraint (33) on x, we can obtain a value for the limit on x as a function of the quantity $w\sqrt{kM_5}$: the associated numerical results are respectively (the necessary condition $w\sqrt{kM_5} = \mathcal{O}(\mathrm{TeV})$ is due to the relation obtained in (44)),

$$x \lesssim \{0.51; 0.78; 1.09; 1.50\}, \quad \text{for } w\sqrt{kM_5} = \{2; 3; 5; 10\}\,\mathrm{TeV}. \qquad (46)$$

Let us comment on the use of the constraint (39) on c for deriving the limit (46) on x. The constraint (39) on c corresponds to the following exact values for the relevant fundamental parameters: $w = 10^{-15}$ and $k/M_5 = 1$. Now, only the associated orders of magnitude are imposed for w and k/M_5 by the condition (12) concerning gauge hierarchy and the constraint on k obtained in (45), respectively. Hence, one may think of considering the constraint (39) on c induced by other values of w and k/M_5 around respectively 10^{-15} and 1. However, the constraint (39) on c possesses a weak dependence on both the parameter w (see end of Sect. 3.3.1) and the ratio k/M_5 (see also (38)). In conclusion, the dependences of the c constraint (39) on w and k/M_5 do not introduce another significant dependence on the fundamental parameters for the x limit (46) (compared to the dependence of x limit (46) on $w\sqrt{kM_5}$).

In summary, we have derived the experimental bound (46) on the fundamental parameter x of the "$+-+$" scenario. The obtained values of this upper bound (46) are valid for $w \sim 10^{-15}$ (necessary for solving the gauge hierarchy problem), $k < M_5$ (condition (9) of validity for the "$+-+$" model) with $k \sim M_5$ (resulting from various relevant constraints) and $M_5^3 \simeq kM_{\rm Pl}^2$ (good approximation of relation (10) characteristic of the "$+-+$" framework), the two latter conditions leading to $k \sim M_5 \sim M_{\rm Pl}$.

From a general point of view, it is particularly interesting to obtain an experimental limit (as in (46)) on the fundamental parameter x in the "$+-+$" framework (with an additional massive bulk neutrino). As a matter of fact, among the five fundamental parameters of the "$+-+$" model including a massive bulk neutrino, namely x, w, k, M_5 and c, only x is really free from the theoretical point of view. In the sense that the four other fundamental parameters undergo the following direct constraints. The warp factor w has to be approximately equal to 10^{-15} if the gauge hierarchy question is to be addressed (see (12)). The AdS_5 curvature k must be of the same order of magnitude as the fundamental Planck mass M_5 in order to avoid the appearance of a hierarchy between energy scales. The value of gravity scale M_5 is restricted via the formula (10) (given in a good approximation by (43)), which is dictated by the "$+-+$" theory, to be a known function of the other fundamental parameters x, w and k (or equivalently L_-, L_+ and k). Finally, the quantity c, which parameterizes the amplitude of 5-dimensional neutrino mass m (see (17)), fixes the neutrino mass scales, and can thus be constrained by considering their realistic value (see Sect. 3.3.1).

(3) Consequences of x constraint for other parameters/quantities: The parameters x (defined by (14)) and w (defined by (11)) of the "$+-+$" scenario can be replaced by the theoretically equivalent parameters L_- and L_+ (defined in Sect. 2.2). Therefore, the experimental bound on x obtained in (46) together with the condition (12) ($\Leftrightarrow$ (13)) on w concerning the gauge hierarchy give rise to the approximative expression for L_- and L_+:

$$L_- \sim L_+ \sim 34/k. \qquad (47)$$

From another point of view, the experimental constraint (33) on the parameter x (illustrated in Fig. 1), which results from a study on the number of neutrino generations, induces in particular a lower bound on the KK mass $m_{\rm KK}^{(1)}$ for the first excited mode of the bulk neutrino since this mass depends on x through formula (21). Following the presentation of Fig. 1, this lower bound is shown on Fig. 2 as a function of the parameter c as the limit (33) on x depends on c. We see on Fig. 2 that this lower bound on $m_{\rm KK}^{(1)}$, once combined with the bound (39) on c (due to experimental constraints on the neutrino masses), gives rise to the conservative bound

$$m_{\rm KK}^{(1)} \gtrsim 1.6\,{\rm TeV}. \qquad (48)$$

(4) Domain of validity for the obtained constraint on x: Let us consider the constraint (33) on x which is illustrated in Fig. 1. This constraint was obtained from considerations on the Z^0 width measurements in the case where only the lightest neutrino eigenstate ν_1 has a mass smaller than half the Z^0 mass (see Sect. 3.1.1). Therefore, this constraint holds in the region of parameter space where only one neutrino eigenstate has a mass smaller than half the Z^0 mass. This region is shown in Fig. 3. It corresponds to the region in which all neutrino mass eigenvalues except

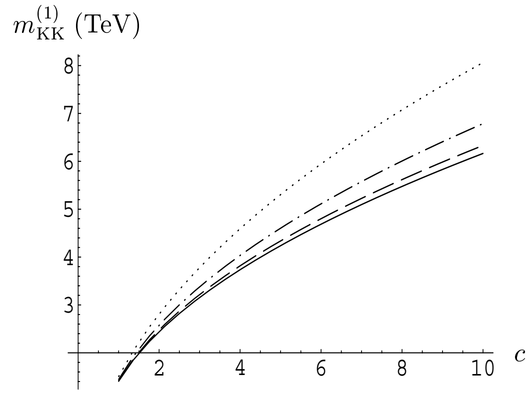

Fig. 2. Mass (21) for the first neutrino KK excitation as a function of the parameter c. The parameter x in mass expression (21) has been set to its limit (33). The quantity $w\sqrt{kM_5}$ entering (33) has been fixed to 2 TeV (dotted line), 3 TeV (dot-dashed line), 5 TeV (dashed line) and 10 TeV (plain line). The choice of taking systematically $w\sqrt{kM_5} = \mathcal{O}({\rm TeV})$ is motivated by the relevant condition obtained in (44). Furthermore, we have considered the case where $k = M_5$ (recall that having $k \sim M_5$ avoids the presence of a new typical energy scale value) so that the quantity $w\sqrt{kM_5}$ is equal to the product wk involved in KK mass expression (21). Domains lying below the curves are ruled out (see (21) and (33))

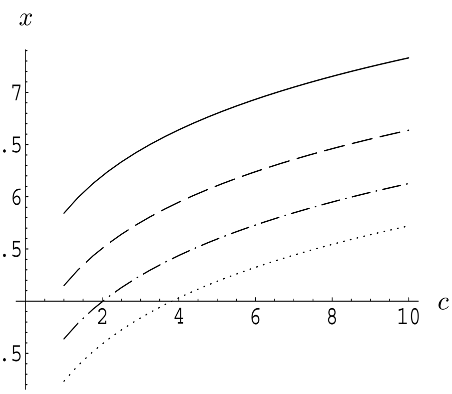

Fig. 3. Values of parameter x (domains below the curves) corresponding to the situation where only one neutrino mass eigenvalue is smaller than half the Z^0 boson mass. Those values are presented as a function of parameter c, for wk equal to 2 TeV (dotted line), 3 TeV (dot-dashed line), 5 TeV (dashed line) and 10 TeV (plain line). We have also set the ratio k/M_5 at the physically relevant value of one so that each of the present curves (obtained for a fixed value of wk) can be associated to a curve in Fig. 1 (obtained for a fixed value of $w\sqrt{kM_5}$). This is useful as the domains shown here (below the curves) represent the regions of validity for the upper bounds on x given in Fig. 1

the smallest one, namely $m_{\nu_2}, m_{\nu_3}, \dots$ (see footnote 11), are larger than half the Z^0 mass. We mention that those eigenvalues are approximatively given by $m_{\nu_2} \simeq m_\nu^{(1)}$ and $m_{\nu_i} \simeq m_{\rm KK}^{(i-1)}$ $[i \geq 3]$ (see the mass definitions in Sect. 2.3) for $c \in [1, 10]$, $wk \in [2, 10]$ TeV and $x \gtrsim 4$.

We deduce from Figs. 1 and 3 that we have only excluded intermediate values of x, or more precisely that the obtained range of values is

$$x < \mathcal{O}(1) \text{ or } \mathcal{O}(4) \leftrightarrow \mathcal{O}(6) < x. \tag{49}$$

We discuss now the scenario where there exist several neutrino mass eigenvalues smaller than half the Z^0 mass, a scenario which arises within the regions of parameter space lying above the curves shown in Fig. 3. For instance, let us consider the simplest case with two neutrino mass eigenvalues smaller than half the Z^0 mass. Then, the admixture $\cos^2\theta_\nu$ introduced in Sect. 3.1.1 would be now defined as

$$\cos^2\theta_\nu = \alpha_1 U_{01}^2 + \alpha_2 U_{02}^2, \tag{50}$$

instead of (29). The suppression factor α_i $[i = 1, 2]$ $(0 < \alpha_i \leq 1)$ quantifies the phase space suppression of the Z^0 decay rate into neutrinos of mass m_{ν_i} (c.f. footnote 11) relative to the Z^0 decay rate into massless neutrinos $\Gamma^{\rm th}(Z^0 \to \bar\nu_{\rm L}\nu_{\rm L})$ (which enters (25)). The suppression factor α_1 is taken to be equal to 1 in a good approximation since the mass m_{ν_1} of lightest neutrino eigenstate ν_1 is negligible compared to the Z^0 mass (see (35)) so that the associated phase space suppression is also negligible. In order to deduce a constraint on the parameters of the "$+-+$" model from the bound (28) on the angle θ_ν due to Z^0 width measurements, one has to express θ_ν (defined now by (50)) in terms of those parameters. This is far from being trivial for the two following reasons. First, the squared matrix element U_{02}^2 entering (50) involves an unknown sum (see (B.5)) of Bessel function roots (via the KK masses $m_{\rm KK}^{(m)}$ $[m \geq 1]$ given in (21)–(22)). Secondly, the factor α_2 and squared matrix element U_{02}^2 (see (B.5)) involve the eigenvalue m_{ν_2}, of the infinite neutrino mass matrix (24), which has to be expressed in terms of the mass matrix elements. Hence, in the domains of parameter space situated above the curves of Fig. 3, it turns out to be difficult to derive analytically a constraint on the parameter x from the bound (28) due to Z^0 width measurements. A numerical approach does not seem easier since the definition of admixture $\cos^2\theta_\nu$ depends on the number of neutrino mass eigenvalues smaller than half the Z^0 mass (as well exhibited by (50)) so that the computation of the constraint on x depends strongly on the region of parameter space studied. Nevertheless, one can predict that this constraint on x, issued from the bound (28) on θ_ν, is less severe than the constraint presented in Fig. 1 (obtained in the domains below the curves of Fig. 3) since there are more positive contributions to the admixture $\cos^2\theta_\nu$ (compare (29) with (50) for example).

(5) Comparison of the bound placed on x with existing bounds: We now compare our experimental bound on x obtained in Fig. 1 (in the parameter spaces illustrated on Fig. 3) with the other experimental bounds on x which have already been derived in the literature [13, 14] within the "$+-+$" framework. These other bounds on x have been obtained by considering KK excitations of the graviton. More precisely, the authors of [13, 14] have placed a constraint on x by requiring that the contribution of resonant graviton KK state production to the SM process $e^+e^- \to \mu^+\mu^-$ is not visible at leptonic colliders. Furthermore, another bound on x has been deduced from the condition that the exchange of graviton KK modes does not induce noticeable corrections to the Newton law (which is tested experimentally) [13, 14].

First, we consider our upper bound (valid for $x \lesssim 4.9$):

$$x \lesssim 0.3, \tag{51}$$

obtained (see Fig. 1) for the typical value $c = 3$, and

$$w\sqrt{kM_5} = 2\,\text{TeV}. \tag{52}$$

Let us also consider, for example, the precise warp factor value (the bound (51) is valid for $w \sim 10^{-15}$)

$$w = 4.5\,\mathrm{e}^{-35} = 2.84\ 10^{-15}. \tag{53}$$

For values of x and w given respectively by (51) and (53), the property (15) of the "$+-+$" scenario reads in a good approximation $M_5^3 \simeq kM_{\rm Pl}^2$, a relation which leads together with (52) and (53) to

$$k = 3.82\ 10^{17}\,\text{GeV}. \tag{54}$$

To summarize, our bound (51) on x holds for the values of fundamental parameters w, k and M_5 given respectively by (53), (54) and (10) ($\Leftrightarrow$ (15)). For the same values of parameters w, k and M_5, the bound on x, coming from considerations on graviton KK state production at colliders, is [13, 14]

$$x \in [0, 1.2]\mathrm{U}[2.3, 3.1], \text{ or } x \gtrsim 4.2, \tag{55}$$

while the bound due to possible modifications of gravity reads [13, 14]

$$x \lesssim 16.6 \tag{56}$$

All those bounds on x are summarized in Table 1 together with other values of the bounds associated to different parameter values[9]. The results presented in (51), (55) and Table 1 show that our constraint on x, obtained from the study of experimental bounds on the neutrino masses, is typically stronger than the existing constraint coming from collider physics.

3.3.4 The case of three lepton flavors

(1) Generalization of the neutrino mass terms: Here, we discuss the realistic situation where there are three families

[9] For the values of warp factor w considered in Table 1, no additional bound on x can be put by considering the production of graviton KK mode at leptonic colliders (via the reaction $e^+e^- \to \gamma +$ light KK mode giving a missing energy signal) [13, 14].

Table 1. Experimental constraints on the parameter x of the "$+-+$" model issued from considerations on neutrino physics [upper line] (c.f. Figs. 1 and 3) and exchange of graviton KK modes [lower line] [13,14] for $c = 3$ and various values of the other theoretical parameters w and k (M_5 being fixed by the characteristic relation (10) or equivalently (15)). The choice of restricting the parameter space to $w \sim 10^{-15}$ and $k \sim M_{\rm Pl}$ is motivated, respectively, by the gauge hierarchy question (see Sect. 2.2) and the condition $k \sim M_5$ (leading together with (15), or equivalently (43), to $k \sim M_{\rm Pl}$) under which no new typical energy scale value is introduced. Finally, all parameter values are taken such that the necessary condition $k < M_5$ (see (9)) is well fulfilled

	$w\sqrt{kM_5} = 2\,\text{TeV}$	$w\sqrt{kM_5} = 5\,\text{TeV}$
$w = 4.5\,\text{e}^{-35}$ $= 2.84\ 10^{-15}$	$[k = 3.82\ 10^{17}\,\text{GeV}]$ $x \in [0, 0.3]$ or $x \gtrsim 4.9$ $x \in [0, 1.2]\text{U}[2.3, 3.1]\text{U}[4.2, 16.6]$	$[k = 1.51\ 10^{18}\,\text{GeV}]$ $x \in [0, 0.6]$ or $x \gtrsim 6.3$ $x \in [0, 1.8]\text{U}[3.1, 17.4]$
$w = 10\,\text{e}^{-35}$ $= 6.30\ 10^{-15}$	$[k = 1.15\ 10^{17}\,\text{GeV}]$ $x \in [0, 0.3]$ or $x \gtrsim 4.5$ $x \in [0, 1.1]\text{U}[2.2, 2.6]\text{U}[3.2, 16.5]$	$[k = 4.56\ 10^{17}\,\text{GeV}]$ $x \in [0, 0.6]$ or $x \gtrsim 5.9$ $x \in [0, 1.8]\text{U}[2.8, 17.2]$

of neutrino. First, in this case, the neutrino mass matrix (24) must be modified. As a matter of fact, it is natural to assign three different 5-dimensional masses m_f $[f = e, \mu, \tau]$ (see (16)), and thus three parameters c_f (see (17)), to the three generations of bulk neutrino Ψ_f. Therefore, there are three masses $m^{(n)}_{\nu_f}$ $[f = e, \mu, \tau; n = 0, 1, 2, \ldots]$, mixing the right-handed modes of three bulk neutrinos $\psi^{(n)}_{f\text{R}}$ with the three left-handed SM neutrinos $\nu_{f\text{L}}$, which are associated to the three parameters c_f (see (18)–(20)). While this mass $m^{(n)}_{\nu_f}$, which must enter (24), differs for each family of bulk neutrino state $\psi^{(n)}_{f\text{R}}$ (associated to c_f), it is identical for each family of SM neutrino $\nu_{f\text{L}}$ if one assumes a universal value for the Yukawa coupling parameter λ_5 (see the action (16)). Similarly, there are now three types of KK mass $m^{(n)}_{KKf}$ $[f = e, \mu, \tau; n = 1, 2, 3, \ldots]$, for the excited modes of three bulk neutrinos $\psi^{(n)}_f$, which correspond to the three parameters c_f (see (21)–(22)).

(2) New bound on x: In Sect. 3.3.3, we deduced the bound (46) on the parameter x from the constraints (33) (due to measurements of the Z^0 width: see Sect. 3.1) and (37) (due to data on the neutrino masses: see Sect. 3.2) in the simplified case of one lepton family. Let us discuss the changes of those limits (46), (33) and (37) as one passes to the case of three lepton families.

First, we determine the equivalent of the constraint (33) in the case of three lepton species. By extending the calculation performed in Appendix B to $N_f = 3$ flavors, we obtain the following expression for the neutrino mixing angle θ_ν, which enters the experimental constraint (28):

$$\tan^2\theta_\nu \simeq \frac{1}{N_f} - 1 + \sum_{m=1}^{\infty}\left(\frac{m^{(m)}_{\nu_e}}{m^{(m)}_{\text{KK}e}}\right)^2 + \sum_{n=1}^{\infty}\left(\frac{m^{(n)}_{\nu_\mu}}{m^{(n)}_{\text{KK}\mu}}\right)^2 + \sum_{p=1}^{\infty}\left(\frac{m^{(p)}_{\nu_\tau}}{m^{(p)}_{\text{KK}\tau}}\right)^2. \quad (57)$$

We notice that, for $N_f = 1$ flavor, this formula reduces well to the result (B.7) found in Appendix B. After replacing the masses $m^{(n)}_{\nu_f}$ and $m^{(n)}_{KKf}$ $[f = e, \mu, \tau; n = 1, 2, 3, \ldots]$, entering (57), by their own expression (see the above discussion, (19)–(20) and (21)–(22)), we find (in the same way as in Sect. 3.1.2) we have

$$\tan^2\theta_\nu \simeq \frac{1}{N_f} - 1 + \frac{v^2}{w^2 k M_5}\sum_{f=e,\mu,\tau}\left[\frac{\text{e}^{(2c_f+1)x}}{2(2c_f+1)} + g(c_f)\right], \quad (58)$$

the function g being defined as in (32). The $\tan^2\theta_\nu$ expression (58) for three flavors exhibits the same kind of structure and the same dependence on the fundamental parameters (x, w, k, M_5 and c_f $[f = e, \mu, \tau]$) as the $\tan^2\theta_\nu$ expression (32) for one flavor. Hence, for identical parameter values, one expects the limit (33) for one flavor (value given in Fig. 1), derived from the combination of the constraint (28) on $\tan^2\theta_\nu$ (due to Z^0 width measurements) with the $\tan^2\theta_\nu$ expression, to be of the same order of magnitude as the equivalent limit for three flavors.

Secondly, we comment the constraint (37) in the situation where three lepton species are considered. In such a situation, the experimental limits on absolute physical neutrino masses are of the order of eV like in the case of a unique lepton generation (first one), as shown in Sect. 3.2. By consequence, one expects that, for three generations, imposing these experimental limits (on the three smallest neutrino mass eigenvalues $m_{\nu_{1,2,3}}$) leads to bounds on the three parameters c_f $[f = e, \mu, \tau]$ of same order as the bound on c (given in (38)–(39)) obtained for one generation from the constraint (37) (application of the experimental limit on the smallest neutrino mass eigenvalue m_{ν_1}).

Finally, since the values of the limit (33) (due to Z^0 width measurements) and the limit on c from the constraint (37) (due to the data on the neutrino masses) are expected to remain of the same order of magnitude when one passes to the case of three lepton flavors (see the above discussions), the bound (46) on parameter x, which is deduced from those two limits, should also still be of the same order in the case of three lepton flavors.

Similarly, in the case of three lepton flavors, by determining the parameter space where three neutrino mass eigenvalues are smaller than half the Z^0 mass, one should find a domain of validity for the limit due to Z^0 width measurements that resembles the domain shown in Fig. 3 for the one flavor case. Indeed, the neutrino mass matrix elements have the same dependence on the fundamental parameters in both the cases of one and three flavors.

(3) Characteristic examples: Now, we will give examples of values, within the case of three lepton flavors, for the bound on the parameter x deduced from the constraints due to Z^0 width measurements and experimental limits on the absolute neutrino masses. Those typical examples will confirm the expectation (see above) that this bound on x has the same order of magnitude in the two cases of one and three flavors.

Let us start by considering the simplified scenario where the three parameters c_f $[f = e, \mu, \tau]$ are related, for instance, through the formula

$$c_\tau = 1.5\, c_\mu = (1.5)^2\, c_e, \qquad (59)$$

which reduces the number of degrees of freedom. The hypothesis (59) is motivated by the fact that values, for the three fundamental parameters c_f, of similar orders of magnitude are desirable. Under this assumption (59), requiring that the three smallest eigenvalues $m_{\nu_{1,2,3}}$ of neutrino mass matrix (24) (modified to involve the three c_f parameters) are smaller than the eV scale (see Sect. 3.2) yields the numerical result

$$c_e \gtrsim 1.25 \quad \text{for } k/M_5 = 1 \text{ and } w = 2\,\text{TeV}/M_{\text{Pl}}, \qquad (60)$$

$$c_e \gtrsim 1.29 \quad \text{for } k/M_5 = 1 \text{ and } w = 10\,\text{TeV}/M_{\text{Pl}}, \qquad (61)$$

which is close to the same result obtained in the case of one neutrino generation for similar values of w (see (39) with following text). Then, by taking into account this bound of (60)–(61) on c_e (together with (59)) and applying the constraint (28) (from Z^0 width measurements) on the $\tan^2\theta_\nu$ expression (58) in terms of all the fundamental parameters, we find (with $k = M_5 = M_{\text{Pl}}$ accordingly to characteristic relation (43))

$$x \lesssim \{1.02; 1.00\} \quad \text{for } \; w\sqrt{kM_5} = \{2; 10\}\,\text{TeV}. \qquad (62)$$

The other example of a simplification hypothesis we consider, namely

$$c_\tau = 2.5\, c_\mu = (2.5)^2\, c_e, \qquad (63)$$

corresponds to values of the three c_f parameters more distinct than in the first hypothesis of (59). Under the assumption (63), the constraints from Z^0 width measurements (namely the constraint (28) on $\tan^2\theta_\nu$) and from limits on the neutrino mass scales (given in Sect. 3.2) lead to the same bound on c_e as in (60)–(61) and then, by using expression (58), to (with $k = M_5 = M_{\text{Pl}}$):

$$x \lesssim \{0.47; 0.46\} \quad \text{for } \; w\sqrt{kM_5} = \{2; 10\}\,\text{TeV}. \qquad (64)$$

In conclusion, the characteristic values (62) and (64) for the bound on the parameter x (issued from experimental considerations on neutrino physics), which were derived in the case of three neutrino flavors, are of the same order as the values for the identical bound obtained in the case of one neutrino flavor (see (46)).

(4) Additional experimental constraints: In Sect. 3, we have used some experimental data on neutrino physics (coming from measurements of the Z^0 width and effective neutrino masses) in order to constrain the "$+-+$" model, and in particular to place a bound on the parameter x, within the typical case of one lepton flavor.

In the more precise case of three lepton flavors, one could also use the bounds on the squared neutrino mass differences (Δm^2_{32} and Δm^2_{21}) and the lepton mixing angles (θ_{12}, θ_{23} and θ_{13}), which are derived from the present results of neutrino flavor oscillation experiments [112,114]. Nevertheless, these lepton mixing angles depend on the neutrino mass matrix, which is predicted by the considered "$+-+$" scenario (with additional massive bulk neutrinos), but also on the charged lepton mass matrix, which in contrast is not determined by our scenario. Therefore, in order to use the bounds on these lepton mixing angles for constraining the "$+-+$" model, one should consider a complementary model dictating the flavor structure of the charged lepton masses, which is beyond the scope of our study.

We also mention that, in a detailed approach based on three lepton species, one could also envisage to use the astrophysical and cosmological constraints (like those due to considerations on big bang nucleosynthesis and duration of the supernova 1987A neutrino burst [115]) on the mixing angles between a sterile neutrino and an active SM neutrino (either ν_e, ν_μ or ν_τ), as well as the SNO (salt phase) data [116, 117] on the fraction of sterile neutrino components in the resultant solar ν_e flux at Earth, and the experimental bound on the branching ratio of the flavor violating decay channel $\mu \to e\gamma$ (which is enhanced by the presence of significantly massive sterile neutrinos [118]). As a matter of fact, recall that, within the considered "$+-+$" scenario, the KK excitations of bulk neutrinos (denoted as $\psi_f^{(n)}$ $[f = e, \mu, \tau; n \geq 1]$) behave like sterile neutrinos.

4 The cases of the "$++$" and "$++-$" models

4.1 The "$++$" model

Here, we discuss, within the same philosophy as above, the case of another realization of the paradigm of anomalously light KK excitations: the "$++$" model.

The absence of any "$-$" brane in the "$++$" model protects it against the problem of radion ghost fields. Nevertheless, it turns out that for the construction of such a "$++$" configuration, it is essential to have the AdS_4 geometry on both branes [16, 17].

In the "$++$" model, the warp function $\mathrm{e}^{-\sigma(y)}$ (which determines the metric as shown in (5)[10]) can reach a min-

[10] In the "$++$" context, with our definition (5) of the metric, the warp function $\mathrm{e}^{-\sigma(y)}$ must also depend on the familiar coordinates x^μ $[\mu = 1, \ldots, 4]$ [16, 17].

imum at the point $y = y_0$ lying between the two "+" branes which sit on the two orbifold fixed points at $y = 0$ and $y = L_+$ (where we live) [16,17]. This geometrical feature is also a fundamental characteristic of the "+ − +" model in which the warp function $e^{-\sigma(y)}$ reaches also a minimum at a point $y = L_-$ (see (8)) lying between two "+" branes at $y = 0$ and $y = L_+$ (here, there exists a "−" brane at the extremum $y = L_-$). In this sense, the geometrical configuration of the "++" model mimics that of the "+ − +" model.

This similarity has two main consequences. First, the localizations of the bulk neutrino KK modes, and thus the masses (given by (18)–(20) in the "+ − +" model) mixing those KK modes with SM neutrinos (trapped at $y = L_+$), should be comparable in the "++" and "+ − +" models. Secondly, the KK masses (given in (21)–(22) for "+ − +" and in [35] for "++") for excitations of bulk neutrinos are similar (with identical dependences on the theoretical parameters) in the "++" and "+ − +" models.

Therefore, the whole effective neutrino mass matrix (which involves only these masses, (18)–(20) and (21)–(22), as (24) shows, within the "+ − +" model) is expected to behave similarly in the equivalent parameter spaces of the "++" and "+ − +" scenarios. Hence, in the "++" scenario (for $w = w(k, L_+, y_0) \sim 10^{-15}$), one expects to deduce, from experimental constraints on the neutrinos, limits on the equivalent parameter

$$x = k(L_+ - y_0) \tag{65}$$

of the same order as the limits on x that we have obtained within the "+ − +" framework, namely

$$x < \mathcal{O}(1) \text{ or } \mathcal{O}(4) \leftrightarrow \mathcal{O}(6) < x. \tag{66}$$

4.2 The "+ + −" model

Let us finally study the third multi-brane RS extension, namely the consistent "+ + −" model. Recall that the "+ + −" model (see Sect. 1) consists of a "+" and a "−" brane placed at the two orbifold fixed points $y = 0$ and $y = \pi R_c$ respectively, the SM fields being confined on a second "+" brane (at $y = L_+$) which moves freely in between ($0 < L_+ < \pi R_c$). Hence, this model does not contain any freely moving "−" brane and thus does not give rise to the existence of radion ghost fields [119].

We begin by describing the neutrino mass matrix in the "+ + −" framework. The elements $m_\nu^{(m)}$ (mixing the SM neutrinos with bulk neutrino KK excitations) of the neutrino mass matrix (see (24)) are given by [35],

$$m_\nu^{(0)} \simeq \sqrt{\frac{k_1}{M_5}\left(c - \frac{1}{2}\right)}\, w^{c-1/2}\, \upsilon, \tag{67}$$

$$m_\nu^{(n)} \simeq \frac{8\,\zeta_n^{-\,2}}{J_{c+1/2}(\zeta_n^-)} \left(\frac{k_2}{k_1}\right)^{3/2} \sqrt{c}\,\mathrm{e}^{-3x}\,\upsilon \quad [n = 1, 2, 3, \ldots], \tag{68}$$

where k_1 and k_2 are the two curvatures of the bulk (satisfying $k_1 \sim k_2$ but with $k_1 < k_2$), $w = w(k_1, L_+) \sim 10^{-15}$, ζ_n^- is the nth root of $J_{c-1/2}(X) = 0$ and (to be compared with the parameters (14) and (65)),

$$x = k_2(\pi R_c - L_+). \tag{69}$$

We note that the mass $m_\nu^{(0)}$ for the 0-mode of the neutrino given in (67) has the same expression as in the "+ − +" context (see (18)).

The other relevant elements of the neutrino mass matrix (see (24)), namely the KK masses $m_{\mathrm{KK}}^{(m)}$ (for bulk neutrino excitations), read [35],

$$m_{\mathrm{KK}}^{(m)} = \zeta_m^-\, w\, \mathrm{e}^{-(x + k_2 L_+)}\, k_2 \quad [m = 1, 2, 3, \ldots]. \tag{70}$$

We now treat the bound (28) on the angle θ_ν issued from Z^0 width measurements within the "+ + −" framework. The admixture $\cos^2\theta_\nu$ introduced in Sect. 3.1.1 is defined by (see (50) for the definition of α_i and (B.5) for the expression of U_{0i}^2)

$$\cos^2\theta_\nu = \sum_{i=1}^{N} \alpha_i U_{0i}^2, \tag{71}$$

where the index $i = 1, 2, \ldots, N$ labels the N neutrino eigenstates $\nu_1, \nu_2, \ldots, \nu_N$ which have masses smaller than half the Z^0 mass (see footnote 11). For $i \in [1, 2, \ldots, N]$, there exists an index $m_{\min}$ such that for $m > m_{\min}$ the relation

$$\frac{m_\nu^{(m)\,2}}{m_{\mathrm{KK}}^{(m)\,2} - m_{\nu_i}^2}\, \frac{m_{\mathrm{KK}}^{(m)\,2}}{m_{\mathrm{KK}}^{(m)\,2} - m_{\nu_i}^2} > \frac{m_\nu^{(m)\,2}}{m_{\mathrm{KK}}^{(m)\,2}} \tag{72}$$

becomes true (because the ratio $m_{\mathrm{KK}}^{(m)\,2}/m_{\nu_i}^2$ becomes sufficiently large). Now, the sum $\sum_{m=1}^{\infty} m_\nu^{(m)\,2}/m_{\mathrm{KK}}^{(m)\,2}$ is divergent. Indeed, this sum reads (see (68) and (70))

$$\sum_{m=1}^{\infty} \frac{m_\nu^{(m)\,2}}{m_{\mathrm{KK}}^{(m)\,2}} \simeq \frac{64\;c}{w^2}\; \frac{\upsilon^2\;k_2}{k_1^3}\; \mathrm{e}^{2(k_2 L_+ - 2x)} \;\sum_{m=1}^{\infty} \frac{\zeta_m^{-\,2}}{J_{c+1/2}^2(\zeta_m^-)}, \tag{73}$$

and one has $\zeta_{m+1}^{-\,2} > \zeta_m^{-\,2}$ and $J_{c+1/2}^2(\zeta_{m+1}^-) < J_{c+1/2}^2(\zeta_m^-)$. Therefore, we deduce from (72) that all the sums entering (c.f. (B.5)) all the squared matrix elements U_{0i}^2 of (71) diverge, so that all those elements tend to zero. As a consequence, the phenomenological condition (28) on θ_ν (c.f. (71)) cannot be fulfilled within the "+ + −" context.

5 Conclusion

We have studied the paradigm of anomalously light KK excitations through its different realizations, namely the multi-brane RS extensions of type "+ − +", "++" and "+ + −". The considered parameter space corresponds to the domains where the gauge hierarchy problem is effectively solved. We have assumed that massive right-handed neutrinos (added to the SM) propagate in the bulk.

We have shown that the present experimental bounds on the neutrino masses and mixing angles either constrain (see the respective limits (49) and (66) on the geometrical parameters (14) and (65) of the "$+-+$" and "$++$" models) relatively strongly (c.f. Sect. 3.3.3) or exclude (as it occurs for the "$++-$" model) the theoretical realizations of the paradigm.

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Appendix A: Neutrino mass eigenvalues

Within the "$+-+$" model, the method we use in order to obtain the physical neutrino masses[11] m_{ν_i} is to diagonalize the hermitian square of the neutrino mass matrix $\mathcal{M}$ (see (23) and (24)):

$$\mathcal{M}\mathcal{M}^\dagger = U \ \mathrm{diag}(m^2_{\nu_1}, m^2_{\nu_2}, m^2_{\nu_3}, \ldots) \ U^\dagger, \qquad \text{(A.1)}$$

U being the neutrino mixing matrix defined by (30). We note that the Yukawa coupling constant λ_5 (c.f. (16)) has been taken real (see Sect. 2.3) so that the neutrino mass matrix $\mathcal{M}$ does not involve any CP violation phase. The presence of non-vanishing complex phases would not affect our study.

(1) One KK excitation: First, we observe that the mass ratio $m_\nu^{(0)}/m_\nu^{(1)}$, which is equal to $w^{c-1/2}$ (see (18) and (19)), has a typical value much smaller than one. Indeed, the gauge hierarchy problem is solved for $w \sim 10^{-15}$ (c.f. (12)) and the experimental bounds on the neutrino mass imply $c \gtrsim 1.08$ (in a conservative way: see Sect. 3.3.1), which lead to $w^{c-1/2} \lesssim 2\ 10^{-9}$.

In the simplified case where only the first KK excitation of the bulk neutrino is considered, a straightforward calculation, at first order in $m_\nu^{(0)}/m_\nu^{(1)}$, gives us the following expressions for the two squared neutrino mass eigenvalues of $\mathcal{M}\mathcal{M}^\dagger$:

$$m^2_{\nu_1} \simeq m^{(1)\ 2}_{\mathrm{KK}} \ \frac{m_\nu^{(0)\ 2}}{m^{(1)\ 2}_{\mathrm{KK}} + m_\nu^{(1)\ 2}}, \qquad \text{(A.2)}$$

$$m^2_{\nu_2} \simeq m^{(1)\ 2}_{\mathrm{KK}} + m_\nu^{(1)\ 2} \ \left(1 + \frac{m_\nu^{(0)\ 2}}{m^{(1)\ 2}_{\mathrm{KK}} + m_\nu^{(1)\ 2}}\right). \qquad \text{(A.3)}$$

(2) The KK tower: Let us begin by determining the smallest squared neutrino mass eigenvalue $m^2_{\nu_1}$, in the general case of an infinite tower of KK states. The eigenvalues $m^2_{\nu_i}$, entering (A.1), are solutions of the equation

$$\det[\mathcal{M}\mathcal{M}^\dagger - m^2_{\nu_i}\mathbf{1}] = 0, \qquad \text{(A.4)}$$

where $\mathbf{1}$ denotes the identity matrix. After calculation of the determinant, this equation can be rewritten as

$$\left[\sum_{p=0}^{\infty} m_\nu^{(p)\ 2} - m^2_{\nu_i} - \sum_{p=1}^{\infty} \frac{m_\nu^{(p)\ 2} \ m^{(p)\ 2}_{\mathrm{KK}}}{m^{(p)\ 2}_{\mathrm{KK}} - m^2_{\nu_i}}\right] \times \prod_{p=1}^{\infty} (m^{(p)\ 2}_{\mathrm{KK}} - m^2_{\nu_i}) = 0. \qquad \text{(A.5)}$$

Since taking $m^2_{\nu_i} = m^{(p)\ 2}_{\mathrm{KK}}$ leads to a divergence in (A.5), this equation is equivalent to

$$\sum_{p=0}^{\infty} m_\nu^{(p)\ 2} - m^2_{\nu_i} - \sum_{p=1}^{\infty} \frac{m_\nu^{(p)\ 2} \ m^{(p)\ 2}_{\mathrm{KK}}}{m^{(p)\ 2}_{\mathrm{KK}} - m^2_{\nu_i}} = 0, \qquad \text{(A.6)}$$

which can be transformed into

$$m_\nu^{(0)\ 2} - m^2_{\nu_i} \left(1 + \sum_{p=1}^{\infty} \frac{m_\nu^{(p)\ 2}}{m^{(p)\ 2}_{\mathrm{KK}} - m^2_{\nu_i}}\right) = 0. \qquad \text{(A.7)}$$

Assuming that an eigenvalue is much smaller than the weakest squared KK mass $m^{(1)\ 2}_{\mathrm{KK}}$, one can deduce its expression at leading order from (A.7) and the result is

$$m^2_{\nu_1} \simeq \frac{m_\nu^{(0)\ 2}}{1 + \sum_{p=1}^{\infty} \frac{m_\nu^{(p)\ 2}}{m^{(p)\ 2}_{\mathrm{KK}}}}. \qquad \text{(A.8)}$$

This eigenvalue would be the smallest one since the others are larger than $m^{(1)\ 2}_{\mathrm{KK}}$ (as we will see in (A.12)). In fact, the squared mass (A.8) is effectively the weakest eigenvalue because the hypothesis made to derive it, namely,

$$m^2_{\nu_1} \ll m^{(1)\ 2}_{\mathrm{KK}}, \qquad \text{(A.9)}$$

constitutes a good approximation in our framework (see the bounds on m_{ν_1}, in (35), and on $m^{(1)}_{\mathrm{KK}}$, in (48) and Fig. 2).

We discuss now the other squared neutrino mass eigenvalues $m^2_{\nu_i}$ $[i \geq 2]$. At leading order in $m_\nu^{(0)}/m_\nu^{(1)}$, (A.6) becomes

$$m^2_{\nu_i} \left(1 + \sum_{p=1}^{\infty} \frac{m_\nu^{(p)\ 2}}{m^{(p)\ 2}_{\mathrm{KK}} - m^2_{\nu_i}}\right) = 0. \qquad \text{(A.10)}$$

The solution $m^2_{\nu_i} = 0$ of this equation corresponds, at leading order in $m_\nu^{(0)}/m_\nu^{(1)}$, to the eigenvalue $m^2_{\nu_1}$ given by (A.8). The other solutions $m^2_{\nu_i}$ $[i \geq 2]$ of (A.10) verify the relation

$$\sum_{p=1}^{\infty} \frac{m_\nu^{(p)\ 2}}{m^{(p)\ 2}_{\mathrm{KK}} - m^2_{\nu_i}} = -1. \qquad \text{(A.11)}$$

For each solution $m^2_{\nu_i}$ $[i \geq 2]$ of (A.11), it is clear that at least one term of the involved sum must be negative. Strictly speaking, for any $i \geq 2$, there exists at least one index p such that $m_\nu^{(p)\ 2}/(m^{(p)\ 2}_{\mathrm{KK}} - m^2_{\nu_i}) < 0$, or equivalently $m^{(p)\ 2}_{\mathrm{KK}} < m^2_{\nu_i}$. We thus conclude that all the eigenvalues (except the smallest one) are larger than the weakest squared KK mass, namely,

$$m^2_{\nu_i} > m^{(1)\ 2}_{\mathrm{KK}} \qquad [i = 2, 3, 4, \ldots]. \qquad \text{(A.12)}$$

[11] The indexes i of the physical neutrino masses m_{ν_i} are chosen such that $m_{\nu_1} < m_{\nu_2} < m_{\nu_3} \ldots$

Appendix B: Neutrino mixing angle

In this appendix, we derive the expression of the neutrino mixing angle θ_ν, defined in (29), as function of the elements entering neutrino mass matrix $\mathcal{M}$ (c.f. (24)) within the "$+-+$" scenario.

The definition (29) of the neutrino mixing angle θ_ν involves the unitary matrix U. The definition (30) of U can be expressed in a more explicit way as

$$\psi^\nu_{\mathrm{L}n} = \sum_{i=1}^{\infty} U_{ni}\ \psi^{\mathrm{phys}}_{\mathrm{L}i} \qquad [n = 0, 1, 2, \ldots], \qquad \text{(B.1)}$$

where the index $i = 1, 2, 3, \ldots$ corresponds to the index of mass eigenstates ν_i (see Sect. 3.1.1) and to the index of the physical neutrino masses m_{ν_i} (see footnote 11). The ith vector U_{ni} represents the eigenstate associated to the eigenvalue $m^2_{\nu_i}$ of $\mathcal{M}\mathcal{M}^\dagger$, and it is clear from (A.1) that for each i value (recall that $i = 1, 2, 3, \ldots$) one has

$$\sum_{n=0}^{\infty} (\mathcal{M}\mathcal{M}^\dagger - m^2_{\nu_i}\mathbf{1})_{mn} U_{ni} = 0 \qquad [m = 0, 1, 2, \ldots]. \quad \text{(B.2)}$$

After replacing $\mathcal{M}$ by its expression (24), in (B.2), we obtain the following system of equations:

$$\left(\sum_{p=0}^{\infty} m_\nu^{(p)\ 2} - m^2_{\nu_i} \right) U_{0i} + \sum_{n=1}^{\infty} m_\nu^{(n)} m_{\mathrm{KK}}^{(n)} U_{ni} = 0, \quad \text{(B.3)}$$

$$m_\nu^{(m)} m_{\mathrm{KK}}^{(m)}\ U_{0i} + (m_{\mathrm{KK}}^{(m)\ 2} - m^2_{\nu_i}) U_{mi} = 0 \qquad [m = 1, 2, 3, \ldots]. \qquad \text{(B.4)}$$

The normalization condition for the ith vector U_{ni}, namely $\sum_{n=0}^{\infty} U^2_{ni} = 1$ $[i = 1, 2, 3, \ldots]$, leads, together with (B.4), to the following analytical expression for U^2_{0i}:

$$U^2_{0i} = \left[1 + \sum_{m=1}^{\infty} \frac{m_\nu^{(m)\ 2}\ m_{\mathrm{KK}}^{(m)\ 2}}{(m_{\mathrm{KK}}^{(m)\ 2} - m^2_{\nu_i})^2} \right]^{-1}. \qquad \text{(B.5)}$$

Therefore, a good approximation of the squared matrix element U^2_{01} associated to the smallest squared neutrino mass eigenvalue $m^2_{\nu_1}$, which verifies (A.9), is

$$U^2_{01} \simeq \left[1 + \sum_{m=1}^{\infty} \left(\frac{m_\nu^{(m)}}{m_{\mathrm{KK}}^{(m)}} \right)^2 \right]^{-1}. \qquad \text{(B.6)}$$

Finally, (B.6) and (29) allow one to obtain the wanted expression for the neutrino mixing angle θ_ν:

$$\tan^2 \theta_\nu \simeq \sum_{m=1}^{\infty} \left(\frac{m_\nu^{(m)}}{m_{\mathrm{KK}}^{(m)}} \right)^2. \qquad \text{(B.7)}$$

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